On Mathematical Simulation

and its analogous statistical simulation method

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When encountering the topic of mathematical simulation, often we see it in the form of ”already-solved” mathematical formulation, embedded a bit of random process and probabilistic estimation, then run it over. One example of such is the Ising model, or the Monte Carlo method of random probabilistic simulation, in which the numerical estimation is defined already.

But where do our topic, and our research, there seems to be more than such. Can we set up a simulating ”environment” to then solve the problem, rather than fiddling around, and attempting to solve it either numerical or else? Can we run the simulation in the form of ”random evolution”, and extract information of it afterward the process, rather than the condition to have either a near-analytical solution, or an iterative numerical solution?

Normally, mathematical modelling, as well as mathematical simulations, formulate ”belief” of a system into a set of mathematical rules, of which then is put upon test, over a population of different agents and ensembles. In here, the process, as well as the simulation is deterministic. The non-deterministic nature, if arise, can be taken into account as certain probabilistic behaviour unseen in consideration.

To this end, let’s take an example of our problem. Suppose that we are assigned a quantum mechanical system $Q(\Psi, \hat{H})$ of which the information of the entire system is encoded by $\Psi(x, t)$. This system evolutional nature is enforced by the Schrodinger equation, which reads:

$$i\hbar\frac{\partial\Psi}{\partial t} = \hat{H}\Psi \tag{1}$$

which enforces certain time-evolution path to the system itself, that all the ”object” in such system must obey. Usually, the Hamiltonian operator must be specified of its internal potential, $V(x)$, as the generalized proxy of the old ”force-action” in classical formulation.

Now, usually, we have to solve the Schrodinger equation, either numerically by approximation, or a closed-form expression of state. However, what if we don’t solve it, at all? Notice that what are we doing, are actually, if in term of quantum simulation, **stochastic process simulation**, of which lies in the domain of time-evolutional system, and is, in surprisingly quite a lot of sense, **non-deterministic**. A deterministic system has the condition that any given ”axis of expression” of the given system must be in an explicit sets of equations and relations. A stochastic process, however, the evolution is not explicit, and rather partially probabilistic, in such way that the time-direction has certain dependences toward the previous state.[1]

However, let's take it a bit further. Remember that the Schrodinger equation encodes the way that the system "evolute", given either the finite resource (stationary case) or variable resource ensemble (different energy state), while specifying changes through the still dynamic wavefunction. So, in essence, wavefunction is dynamic no matter what, but depends on the Schrodinger equation, of which itself is a second-order PDE, with, usually in reality, no closed-form solution given. But, all of them are **time-evoluting**, give or take, under different set of evolution axis. What if we can do a testing, loosely called **stochastic time-restricted evolutional testing**?

The idea is simply this:

1. Specify the system in term of macroscopic rules: The resources r, some more macroscopic feature set $\{P\}$ the certain evolutional behaviour in two forms: free-evolution axis $F(P, t)$, and restricted evolution axis, $Rs(P, t)$; the system condition S_F, the interval evolutional rule $I(P, t)$, and two masks $M = \{t_{acc}, t_{rej}\}$. There is also a blank space, for any additional information applied on such system.

2. Start by giving the simulation all the given information, except $Rs(P, t)$.

3. Start the simulation by the agent-system rule with variable ensemble of initial agent. Keep it random evolution in certain time interval (continuous or discrete) $[t_1, t_k]$.

4. Within such interval, process like a Markov decision process - those that are qualified of $Rs(P, t)$ in the not so explicit, but proportionally equaivalent form, would be accepted. The others would be "rejected" - by creating a 'cloud of non-preference' around the evolutional path that this agent made along the way. All ensemble to this point, will be apparent of the knowledge of this 'non-preference', after time $t + 1$ that will be in effect.

5. Repeat this for increment t, and averaging out the entire ensemble result.

This seems varied - more like *random walk*, but also feels like it belongs to certainly pathway finding solution. However, notice that for this kind of thing, the limit is that we are testing with variable ensemble - independent of each other. Given such system where individual agents have effects on others, it is unlikely that the above formulation is complete for the operation to actually make sense, not even running of anything. Still, quite an idea. Should we test this so far?

Reference

1. Introduction to Stochastic Process - MA636, University of Kent.

[1] The line is kind of blurred out, certainly from quite the perspective, since there is also the notion of **deterministic chaos**, or chaos for short - extremely sensitive deterministic system on initial conditions